Symmetrical components theory for calculation of fault current using simple drawings

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***Abstract*—This paper proposes a simplified approach for the computation of fault current during unbalanced conditions which include single phase to ground fault, phase to phase fault and double phase to ground fault. Complex mathematical equations based on phasor methods are used in classical technique for calculating unbalanced fault current. Though this method offers accurate results, its application in real-time fault analysis is often mathematically cumbersome and time-consuming. To overcome this limitation, this paper proposes an intuitive approach utilizing simple drawing of symmetrical component to analyse the behaviour of electrical power system during an unbalanced fault. The proposed method covers all three scenarios of unbalanced fault in detail and explains all calculation processes step by step without performing exhaustive mathematical calculations. Furthermore, this method extends to account for the impact of transformers, particularly delta/star configurations, which introduce phase shifts and trapping of zero-sequence currents. Through the simple drawing of symmetrical components in place of phasor computation, this technique offers power engineers a simple, yet efficient technique for the analysis of fault currents under unbalanced conditions.**

Keywords—unbalanced fault analysis, simplified approach, symmetrical component, simple drawing

I. INTRODUCTION

Accurate analysis of fault currents in power systems is essential for designing reliable protection schemes and maintaining system stability. Among various fault types, unbalanced faults such as single phase-to-ground (SPG), phase-to-phase (PP), and double phase-to-ground (DPG) faults present greater complexity due to the asymmetry they introduce.

These faults are typically analysed using the method of symmetrical components, which decomposes unbalanced phasors into three sets of balanced components: positive, negative, and zero sequences [1]. Although this method provides precise mathematical modelling, it often involves extensive use of phasor algebra, which can limit intuitive understanding and increase complexity in fault studies.

To improve clarity and accessibility in both educational and practical applications, graphical methods have been introduced to interpret the behaviour of symmetrical components. These approaches provide visual insight into the relationships among sequence currents and boundary conditions without relying on algebraic calculations [2], [3].

This paper presents a graphical method for analyzing unbalanced faults in three-phase systems. It focuses on using vector-based diagrams to derive fault current magnitudes and directions through simple drawings. The method is applied to SPG, PP, and DPG faults under various system configurations, including the presence of a delta-wye transformer. By simplifying the interpretation of symmetrical components, this approach enhances understanding and serves as a useful alternative to traditional phasor-based analysis.

II. THEORETICAL BACKGROUND

In balanced three-phase systems, analysis is straightforward due to the uniformity in phase magnitudes and 120° phase separations. However, unbalanced faults introduce asymmetries that require more advanced techniques. The method of symmetrical components, first introduced by Fortescue [1], enables the decomposition of unbalanced phasors into three balanced sets: positive, negative, and zero sequences. The positive sequence consists of phasors of equal magnitude spaced 120° apart in the original phase sequence. The negative sequence also has 120° spacing but in the reverse order. The zero sequence consists of three phasors equal in both magnitude and angle.

This decomposition allows the use of independent sequence networks for analysis. Depending on the fault type, these sequence networks are interconnected differently:

- SPG faults involve positive, negative, and zero sequence networks in series.
- PP faults involve only the positive and negative sequence networks.
- DPG faults involve all three, but with a different interconnection than SPG [2].

The graphical method used in this paper is grounded in the same principles as the symmetrical components method but provides a more intuitive visual framework. Rather than solving equations algebraically, the sequence relationships and boundary conditions are represented through simple drawings. This approach not only simplifies the interpretation of sequence currents but also reinforces conceptual understanding.

III. METHODOLOGY

This section presents the methodology for determining fault currents in unbalanced three-phase systems using a graphical approach. Rather than relying on phasor algebra, the approach uses simple drawings based on the principle of symmetrical components and known boundary conditions. These visual tools represent the interaction of sequence currents under faulted conditions and are applied to three common fault types: single phase-to-ground (SPG), phase-to-phase (PP), and double phase-to-ground (DPG). The method is also extended to systems with delta-wye transformers to illustrate its versatility.

A. Single Phase-to-Ground (SPG) Fault

Consider a single phase-to-ground fault occurring at the red phase. The boundary conditions for the phase currents are given by:

$$I_R = I_{fault},\ \ I_Y = I_B = 0$$

where I_R, I_Y, and I_B are the phase currents for the red, yellow, and blue phases, respectively.

Using symmetrical components, the conditions from the boundary can be expanded as:

$$I_R = I_{R1} + I_{R2} + I_{R0} = I_{fault} \tag{1}$$

$$I_Y = I_{Y1} + I_{Y2} + I_{Y0} = 0 \tag{2}$$

$$I_B = I_{B1} + I_{B2} + I_{B0} = 0 \tag{3}$$

where the subscripts 1, 2, and 0 represent positive, negative, and zero sequence components, respectively.

From (2) and (3), the non-faulted phases can be rewritten as:

$$I_{Y1} + I_{Y2} = -\,I_{Y0} \tag{4}$$

$$I_{B1} + I_{B2} = -\,I_{B0} \tag{5}$$

These relationships form the basis for constructing the vector diagram shown in Fig. 1(a). The non-faulted phases Y and B are drawn to satisfy the zero-current condition, while the rotation of the positive and negative sequence components adheres to the principle of symmetrical components.

Fig. 1(b) visually confirms these conditions. For phase Y, the vector representing the sum of I_{Y1} and I_{Y2} points downward (at 6 o'clock), while I_{Y0} points upward (at 12 o'clock). A similar configuration is observed for phase B, validating conditions (4) and (5).

For the red phase, Fig. 1(b) shows that the three components I_{R1}, I_{R2} and I_{R0} are equal in magnitude and direction. As they sum directly to the fault current, this implies a series connection of the positive, negative, and zero sequence networks, as shown in Fig. 1(c).

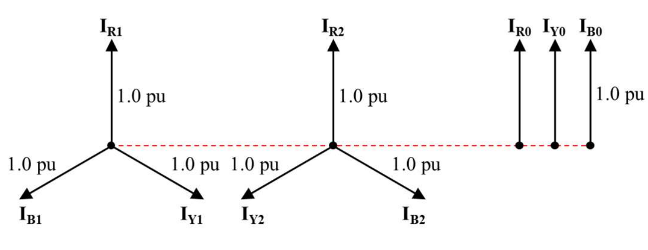
Based on the sequence network diagram in Fig. 1(c), the magnitude of each symmetrical component current is:

$$I_{R1} = I_{R2} = I_{R0} = \frac{V}{Z_1 + Z_2 + Z_0} \tag{6}$$

where V is the pre-fault phase-to-neutral voltage and Z_1, Z_2 and Z_0 are the positive, negative, and zero sequence impedances, respectively.

Since the total fault current is the sum of all three symmetrical components, it is given by:

$$I_{fault} = I_{R1} + I_{R2} + I_{R0} = \frac{3V}{Z_1 + Z_2 + Z_0} \tag{7}$$



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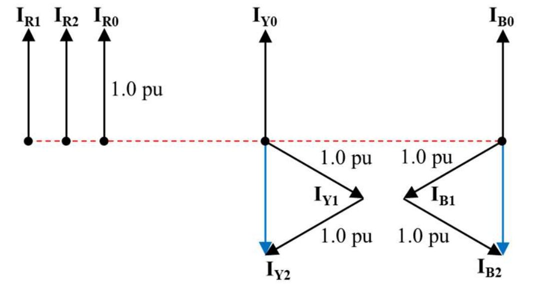

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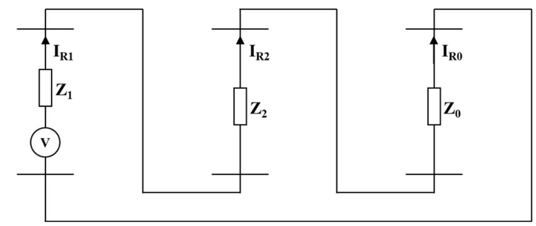


(c)

Fig. 1. Phase-to-ground fault (a) Symmetrical component currents (b) Phase current vectors illustrating the summation of symmetrical components for all three phases. (c) Sequence network impedance diagram for the faulted red phase.

B. Phase-to-phase (PP) Fault

A phase-to-phase fault is considered between the yellow and blue phases, with the red phase un-faulted. The boundary conditions for the phase currents are:

$$I_R = 0,\ \ |I_Y| = |I_B| = |I_{fault}|$$

Since zero sequence current does not exist in a phase-to-phase fault, the boundary conditions using symmetrical components expand to:

$$I_R = I_{R1} + I_{R2} = 0 \tag{8}$$

$$|I_Y| = |I_{Y1} + I_{Y2}| = |I_{fault}| \tag{9}$$

$$|I_B| = |I_{B1} + I_{B2}| = |I_{fault}| \tag{10}$$

From (8), it follows that $I_{R1} = -\,I_{R2}$, meaning the positive and negative sequence currents in the red phase are equal in magnitude but opposite in direction. Based on this condition, the symmetrical component vectors are constructed as shown in Fig. 2(a). The remaining vectors for the yellow and blue phases are drawn according to the principle of symmetrical components.

In Fig. 2(b), the fault currents I_Y and I_B are constructed using (9) and (10). These vectors have equal magnitude of $\sqrt{3} \times I_{R1}$ and flow in opposite directions, with I_Y pointing toward 3 o'clock position and I_B pointing toward 9 o'clock position. This supports the boundary condition $|I_Y| = |I_B| = |I_{fault}|$.

The same figure also illustrates the symmetry in the red phase. To satisfy $I_{R1} = -\,I_{R2}$, the positive and negative sequence networks for the red phase must be connected in series, as shown in Fig. 2(c).

From this sequence network, the red phase positive sequence current is:

$$I_{R1} = \frac{V}{Z_1 + Z_2} \quad (11)$$

The resulting fault current in both yellow and blue phases is then:

$$|I_Y| = |I_B| = \sqrt{3} \text{ x } I_{R1} = \sqrt{3} \text{ x } \frac{V}{Z_1 + Z_2} \quad (12)$$

These currents, as drawn in Fig. 2(b), point towards 3 o'clock and 9 o'clock for I_Y and I_B, respectively.

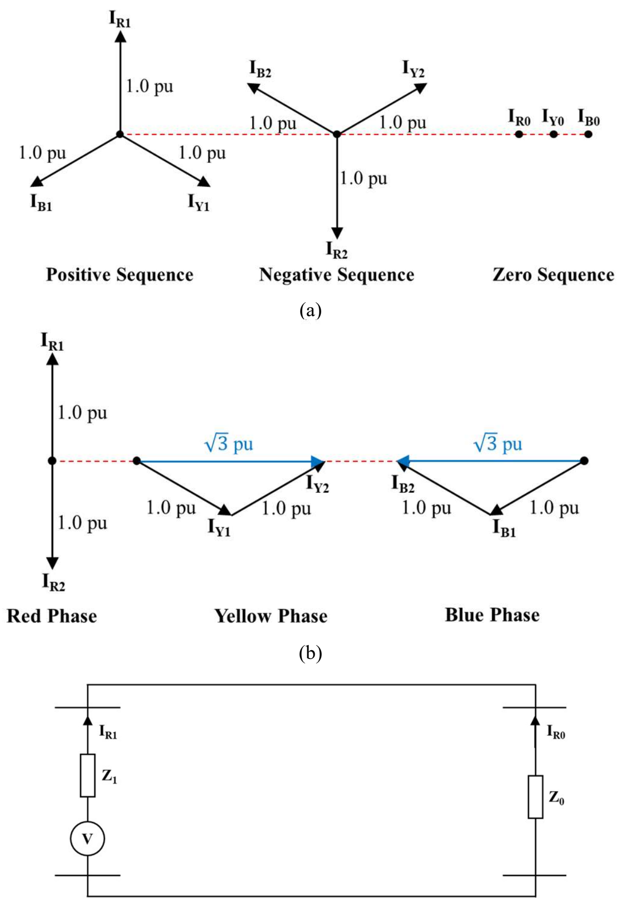


Fig. 2. Phase-phase fault (a) Symmetrical component currents (b) Phase current vectors illustrating the summation of symmetrical components for all three phases. (c) Sequence network impedance diagram for the unfaulted red phase.

C. Double Phase-to-Ground Fault

A double phase-to-ground fault is now considered at the yellow and blue phases, with the red phase remaining unfaulted. Since this fault involves both a phase-to-phase and ground connection, all three symmetrical components, i.e. positive, negative, and zero, are present. The key boundary conditions for the phase currents are given by:

$$I_R = 0, \;\; I_Y + I_B = I_{fault}$$

Using symmetrical components, these can be expanded as:

$$I_R = I_{R1} + I_{R2} + I_{R0} = 0 \quad (13)$$

$$I_Y = I_{Y1} + I_{Y2} + I_{Y0} \quad (14)$$

$$I_B = I_{B1} + I_{B2} + I_{B0} \quad (15)$$

In this fault type, three fault currents are of interest: the individual phase currents I_Y and I_B, and the total fault current $I_Y + I_B$, which flows to ground. The line-side CTs detect I_Y and I_B, while the total ground-return current $I_Y + I_B$ is detected by a neutral CT at the generator or transformer.

Unlike SPG and PP faults, these equations do not yield a unique solution for the symmetrical components. Multiple combinations of I_{R1}, I_{R2}, I_{R0} can satisfy the zero-current condition in (13). One possible solution is visualized in Fig. 3(a) and Fig. 3(b), where the vectors are drawn to satisfy $I_{R1} = -(I_{R2} + I_{R0})$. The ratio between I_{R2} and I_{R0} is determined by their corresponding sequence impedances.

The sequence network for this fault is shown in Fig. 3(c), where the current I_{R1} from the positive sequence source divides into I_{R2} and I_{R0}. The equivalent total impedance is the sum of Z_1 and the parallel combination of Z_2 and Z_0, giving:

$$I_{R1} = \frac{V}{Z_1 + (Z_2 \,||\, Z_0)} \quad (16)$$

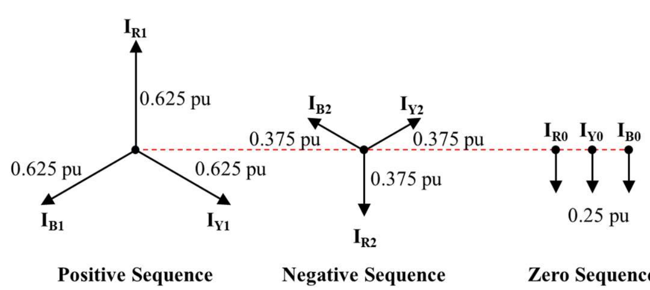
where $\quad Z_2 \,||\, Z_0 = \dfrac{Z_2 \times Z_0}{Z_2 + Z_0}$

Once I_{R1} is known, the current divides between the zero and negative sequence networks based on their relative impedances:

$$I_{R2} = \left(\frac{Z_0}{Z_0 + Z_2}\right) I_{R1} \quad (17)$$

$$I_{R0} = \left(\frac{Z_2}{Z_0 + Z_2}\right) I_{R1} \quad (18)$$

Assuming typical values of $Z_1 = Z_2 = 1.0$ pu and $Z_0 = 1.5$ pu, the positive, negative and zero sequence for red phase is 0.625 pu, 0.375 pu and 0.25 pu respectively. To determine the actual fault currents in the yellow and blue phases, the vectors $I_Y = I_{Y1} + I_{Y2} + I_{Y0}$ and $I_B = I_{B1} + I_{B2} + I_{B0}$ are drawn in Fig. 3(b). Unlike SPG or PP faults, these cannot be calculated by direct addition of symmetrical component magnitudes. Instead, the magnitudes are measured as the length of the vectors where I_Y = length of AB = 0.94 pu and I_B = length of CD = 0.94 pu.



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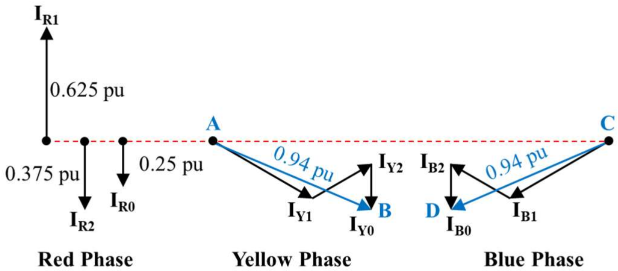

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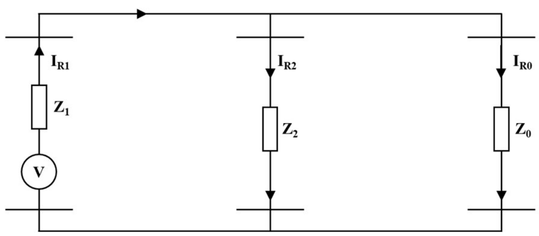


(c)

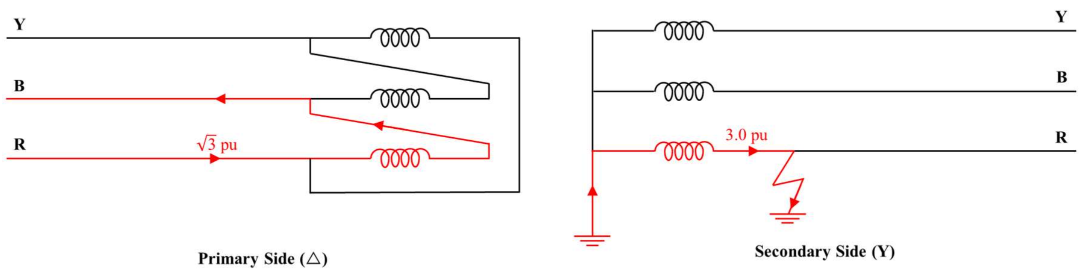
Fig. 3. Double phase-to-ground fault (a) Symmetrical component currents (b) Phase current vectors illustrating the summation of symmetrical components for all three phases. (c) Sequence network impedance diagram for the unfaulted red phase.

D. Single Phase-to-Ground Fault With Trasnformer

A single phase-to-ground fault is now examined with the fault occurring on the star side of a delta-wye (Δ-Y) transformer, as shown in Fig. 4(a). Building on the analysis in Section III.A, the fault current magnitude is taken as 3 pu.

When reflected across the transformer, the fault current undergoes a phase shift due to the Δ-Y configuration. The positive sequence current is shifted 30° in the anticlockwise direction, while the negative sequence current experiences a 30° shift in the clockwise direction. This phase shift is illustrated in Fig. 4(b). The zero sequence current is trapped within the delta winding and is not reflected on the delta side, as the delta connection does not provide a return path for zero sequence components.

The magnitude of the red and blue phase fault currents can then be measured directly from the graphical drawing. As shown in Fig. 4(c), the red and blue phase currents each have an equal magnitude of $\sqrt{3}$ pu, pointing in opposite directions toward the 12 o'clock and 6 o'clock positions, respectively. This indicates that the fault manifests as a double-phase current distribution on the delta side.



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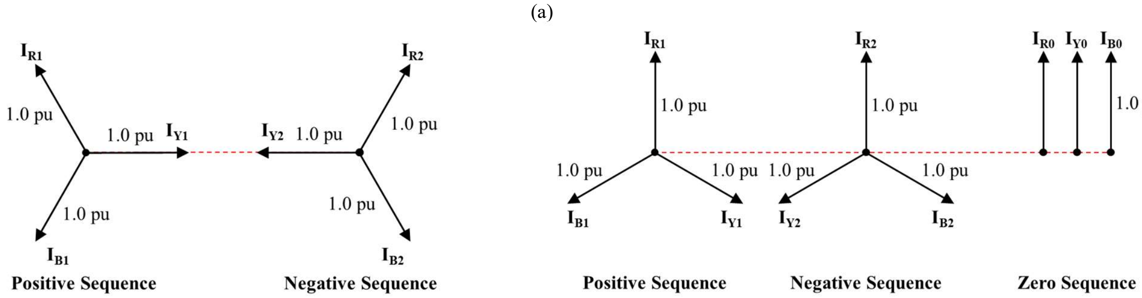

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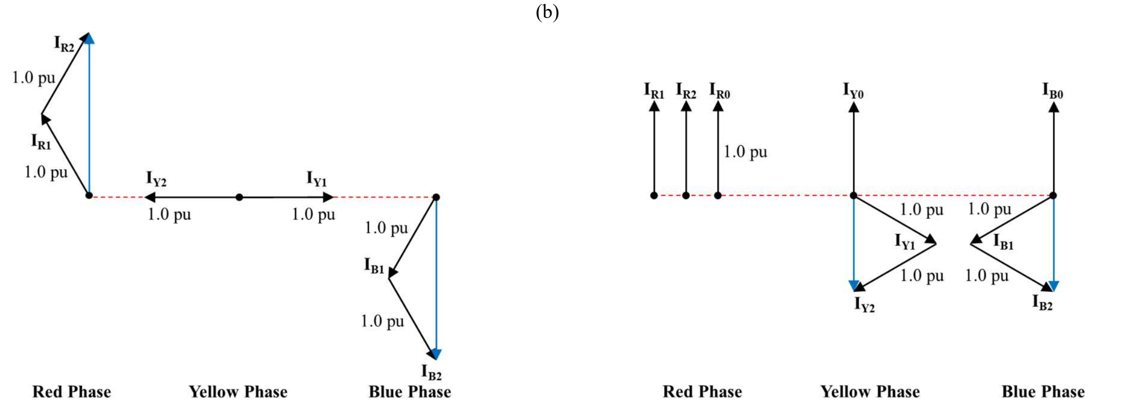

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Fig. 4. Single phase-to-ground fault with transformer (a) Fault current flow in a Δ-Y transformer (b) Symmetrical components currents (c) Phase current vectors illustrating the summation of symmetrical components for all three phases

E. Phase-to-Phase Fault with Transformer

This section extends the analysis in Section III.B by introducing a delta-wye (Δ-Y) transformer into the system. The same phase-to-phase fault between the yellow and blue phases is considered, and the corresponding fault magnitude is taken as $\sqrt{3}$ pu, as previously derived. Graphical method can be applied to determine the fault current on the delta side, taking into account the 30° phase shift introduced by the transformer configuration. This phase shift is illustrated in Fig. 5(b).

The resulting fault current for each phase is obtained graphically. As shown in Fig. 5(c), the yellow phase fault current has a magnitude of 2 pu and points toward the 3 o'clock position. The red and blue phase fault currents each have magnitudes of 1 pu and point toward the 9 o'clock position. These values indicate a three-phase current pattern with a 1:1:2 magnitude ratio on the delta side.

This transformation demonstrates that a phase-to-phase fault of magnitude $\sqrt{3}$ pu on the star side is reflected as an unbalanced three-phase fault on the delta side. The graphical method provides a clear visualization of this transformation.

Fig. 5(a) illustrates the fault current flow and sequence component behaviour through the Δ-Y transformer under this condition.

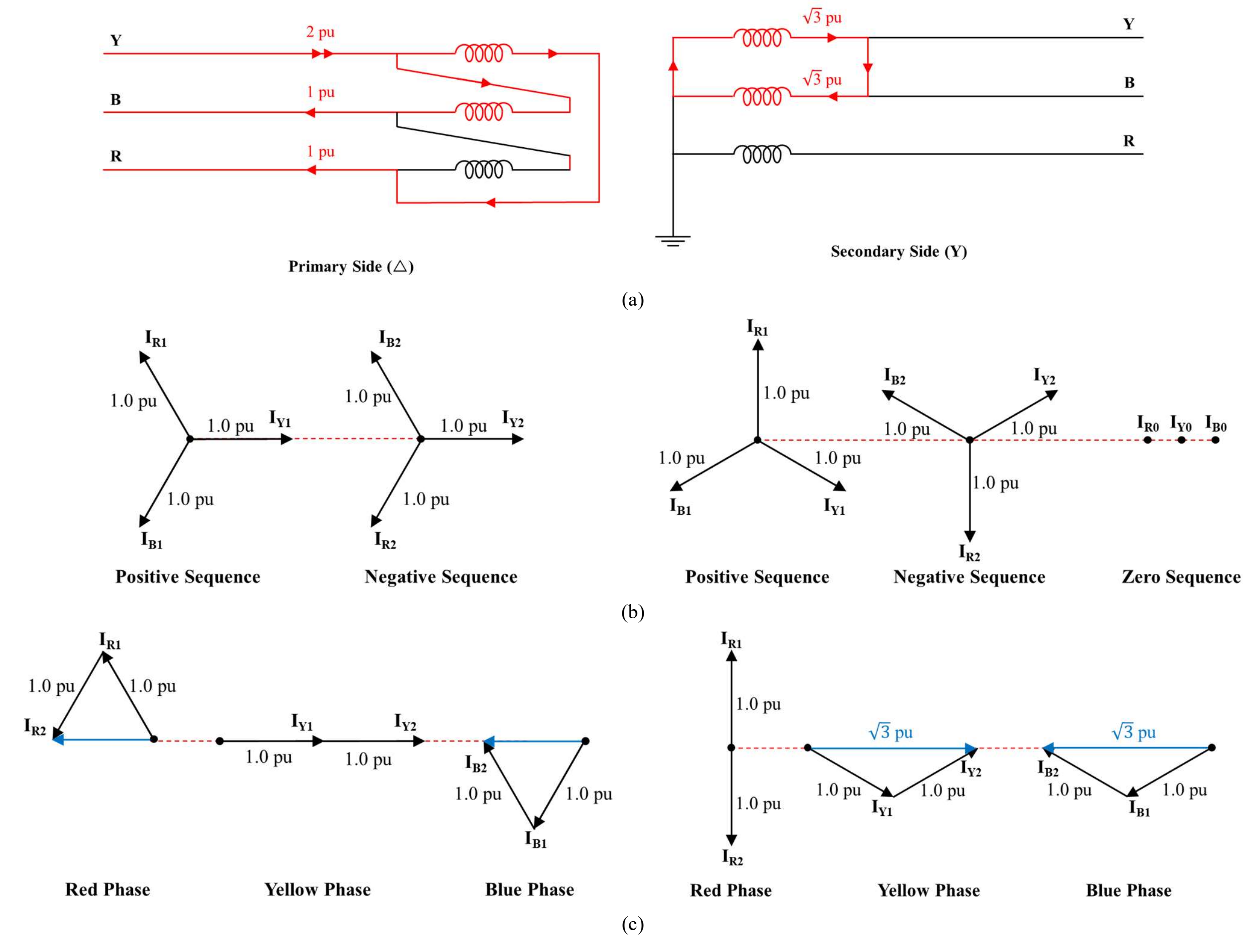

Fig. 5. Phase-to-phase fault with transformer (a) Fault current flow in a Δ-Y transformer (b) Symmetrical components currents (c) Phase current vectors illustrating the summation of symmetrical components for all three phases

F. Double Phase-to-Ground Fault with Transformer

Building upon the double phase-to-ground fault previously analyzed in Section III.C, this section introduces the effect of a delta-wye (Δ-Y) transformer with the fault occurring on the star side, as illustrated in Fig. 6(a). The goal is to determine how the fault currents appear on the delta side using the graphical method, considering the phase shift and zero sequence blocking introduced by the transformer configuration.

As described in previous sections, the positive and negative sequence components undergo 30° phase shifts in opposite directions when passing through the Δ-Y transformer, while the zero sequence component is trapped in the delta winding and does not appear on the delta side. These phase shifts are applied to the symmetrical component diagram in Fig. 3(a), resulting in the transformed sequence diagram shown in Fig. 6(b).

The fault currents are derived using graphical construction, as shown in Fig. 6(c). The yellow phase fault current is found to be 1 pu, while the red and blue phase fault currents are each 0.55 pu. These values reflect the influence of the transformer's configuration and the absence of zero sequence current in the delta winding.

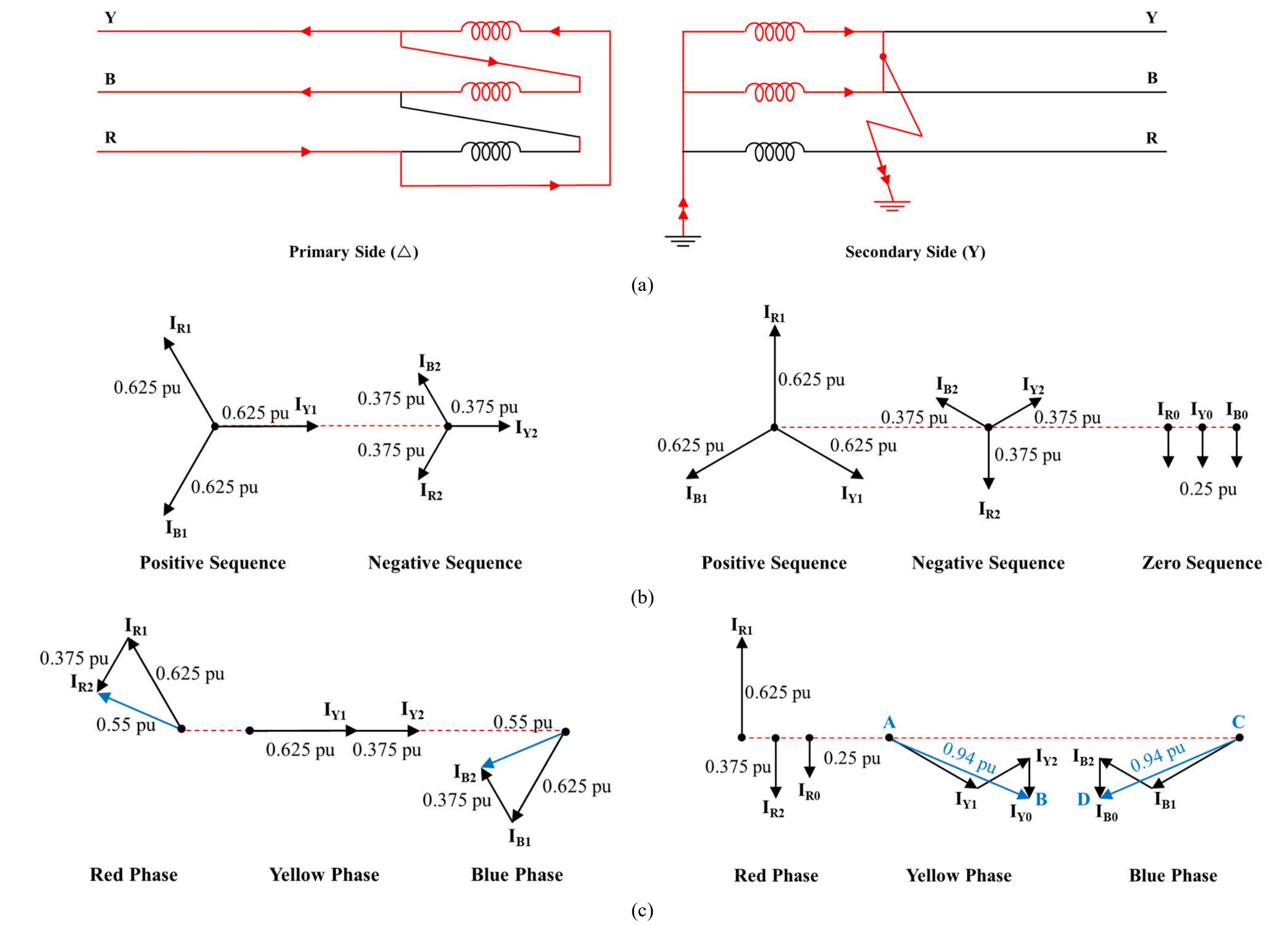

Fig. 6. Double phase-to-ground fault with transformer (a) Fault current flow in a Δ-Y transformer (b) Symmetrical components currents (c) Phase current vectors illustrating the summation of symmetrical components for all three phases

IV. Conclusion

This paper presented a graphical approach to analyze unbalanced faults in three-phase power systems. By relying on the principle of symmetrical components and known boundary conditions, the method enables fault current magnitudes and directions to be determined through simple vector drawings, without the need for phasor algebra.

The approach was applied to three major unbalanced fault types, i.e. single phase-to-ground, phase-to-phase, and double phase-to-ground, and was further extended to include transformer configurations involving delta-wye connections. For each case, graphical construction provided intuitive insights into sequence current behavior and fault current distribution.

Compared to conventional analytical methods, the graphical technique offers a more accessible and visually guided alternative, especially useful in educational settings and initial fault assessment. It also reinforces a deeper understanding of the interaction between sequence networks under various fault conditions.

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